Parallel Graph Summarization for Knowledge Search

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ABSTRACT

Querying heterogeneous and large-scale knowledge graphs are typically expensive. This paper studies a parallel graph summarization framework to facilitate knowledge graph search. (1) We propose a class of *reduced summaries* characterized by graph patterns, which are capable of summarizing entities in terms of their neighborhood similarity up to a certain hop. (2) We study a bi-criteria *diversified summarization* problem. Given a knowledge graph G, it is to discover top-k diversified reduced summaries with maximized quality in terms of both informativeness and diversity. (3) We show that diversified summarization is feasible for large knowledge graphs, by developing a parallel approximation algorithm with quality guarantees. We show that the algorithm is parallel scalable, which ensures the feasibility of summarization in large graphs. Using real-world graphs, we experimentally verify the efficiency of our sequential and parallel summarization algorithms, and query evaluation guided by summarization.

1 INTRODUCTION

Knowledge graphs are routinely used to represent entities and their relationships in knowledge bases [2, 9]. Unlike relational data, real-world knowledge graphs lack the support of well-defined schema and typing system. To search knowledge graphs, a number of query processing techniques are proposed [9, 14, 18, 28]. Nevertheless, it is hard for end-users to specify precise queries that will lead to meaningful answers without prior knowledge of the underlying graph. Searching knowledge graphs is challenging due to the ambiguity in queries, the inherent computational complexity (*e.g.,* subgraph isomorphism [9, 18]) and possible resource constraints (*e.g.,* data allowed to be accessed, response time) [6].

Example 1: Fig. 1 illustrates a sample knowledge graph G of artists and bands. Suppose a music publisher wants to find artists who are experts in two genres (genre), acted in a film, and also collaborated with a band whose manager is located in the same country as the band. This search can be represented as a *graph query* Q [9, 14, 18, 28] as shown in Fig 1. The answer of Q refers to the set of entities typed with an artist in the subgraphs of G that are isomorphic to Q. In this example, T.McGraw is the correct answer for Q.

It is costly to evaluate Q when G is large. For example, the ambiguous label "artist" requires the inspection of all the entities having the type. It is also hard for the users to specify Q without prior knowledge of G. Moreover, subgraph isomorphism is NP-hard.

Observe that graph G can be "summarized" by three small graph patterns P_1, P_2 and P_3, as illustrated in Fig. 1. Each pattern abstracts a fraction of G, by representing a group of entities as a single

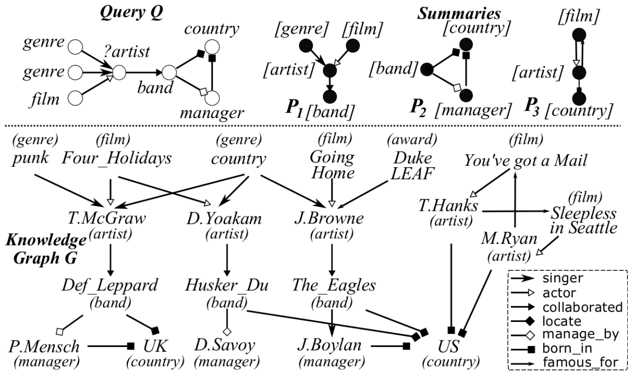


Figure 1: Knowledge graph, summaries and graph query.

node, along with their common neighboring entities in G. For example, P_1 specifies three artists J.Browne, T.McGraw and D.Yoakam in G as a single node artist, who are associated with their band, genre and films, indicating "musicians"; and P_3 distinguishes the artists T.Hanks and M.Ryan who are associated with only film and country (i.e., "actors"). These concise summaries help the users in understanding G without a daunting inspection of low-level entities.

One can further use these patterns as "views" [4, 13] to speed up knowledge search. For example, Q can be correctly answered by only referring to the entities summarized by P_1 and P_2 in G, as all the matches of Q are contained in those entities. □

The above example suggests that graph patterns can benefit knowledge search by suggesting (and can be directly queried as) interpretable "views". Although desirable, computing summaries is nontrivial. Summaries defined by frequent subgraphs [3, 10, 11, 14] can often be an overkill for entities with similar neighbors up to a certain hop. For example, the subgraphs induced by J.Browne and T.McGraw along with their 1 hop neighbors (Fig. 1) *should* be summarized by P_1, despite that they are not isomorphic to each other. Mining summaries is also expensive when G is large.

Contributions. This paper studies a new parallel graph summarization framework to compute diversified summaries for knowledge graph query evaluation.

(1) We review a class of summaries namely, *d-summaries* [21], and introduce reduced d-summaries. A d-summary is characterized by a *d-matching* relation, which encodes a lossy representation of similar entities and their d-hop neighbors. A reduced summary encodes a minimum representation of all equivalent summaries.

We show that d-summary is feasible in practice, by studying the *verification* and *reduction* problems. The verification problem is to check whether a graph pattern is a d-summary. The reduction problem computes a reduced counterpart of a summary. We show that both problems are tractable (Section 2) for d-summaries.

(2) We study a bi-criteria function to quantify the quality of summaries that integrates both informativeness and diversity measures (Section 3). Based on the quality function, we introduce the problem of *diversified graph summarization*. Given a knowledge graph G, integers k and d, the problem is to compute a set of k d-summaries that maximizes the bi-criteria function.

(3) The diversified summarization problem is (not surprisingly) NP-hard. Nevertheless, we show that it is feasible to discover diversified summaries in large graphs, by developing a new parallel diversified summarization algorithm (Section 4). The algorithm has the *parallel scalability*, a guarantee to reduce response time with the increase of processors. This ensures the feasibility of summarization in large graphs by adding processors.

(4) Using real-world knowledge bases and synthetic graphs, we experimentally verify the effectiveness and efficiency of our summarization and query-evaluation algorithms (Section 5). We found that it is feasible to compute diversified summaries over real-world knowledge graphs. Our case studies also verifiy that summary-based querying supports "cross-domain" querying by accessing summaries from multiple knowledge bases.

Related work. We categorize the related work as follows.

Graph summarization. Graph summarization has been studied to describe the data graph with a small amount of information [11, 17, 19, 23, 25, 26]. (1) Graph compression aims to compress graphs within a bounded error by minimizing an information complexity measure [11, 17, 19], e.g., Minimum Description Length (MDL). The algorithm in [11] employs clustering and community detection to describe the data graph with predefined frequent structures (vocabulary) including stars and cliques. (2) Summarization techniques attempt to construct summaries over attributed graphs, where nodes with similar attributes are clustered in a controlled manner using parameters such as participation ratio [25]. (3) (Bi)simulation relation is adoptedto group paths carrying same labels up to a bounded length [26]. It summarizes the entities only when they are pairwise similar, which can be an overkill for knowledge graphs. Entity summarization [23] generates diversified answers for entity search instead of general queries.

Our work differs from these works in the following ways: (1) We introduce *lossy* summaries for knowledge query evaluation, rather than to compress the graphs [11, 17, 19]. (2) We discover summaries to access single graphs rather than for query answers [23, 26], and it can be applied for diversified result summarization. (3) The summaries are measured in terms of both informativeness and diversity, which is more involved than MDL-based measures [17, 19]. (4) In contrast to [17, 25], our summary model requires little parameter tuning effort. In addition, diversified summaries are not addressed in these works.

Parallel graph pattern mining. Frequent subgraph patterns can be mined from a single graph to describe large graphs [3, 11]. Redundancy-aware mining is studied for general patterns [27]. Type-based summarization is applied to facilitate keyword search in RDF graphs [14]. In contrast, we capture summaries in terms of approximate pattern matching rather than strict subgraph isomorphism [3, 14]. Bi-criteria summarization that integrates diversification and informativeness is not addressed in these works.

Parallel algorithms have been developed for pattern mining in terms of subgraph isomorphism, for transactional graph databases [1] or single graph [24]. Vertex-centric models (*e.g.,* Pregel [16] and GraphLab [15]) are developed to parallelize sequential graph query processing. These methods can not be readily applied for diversified summarization in a single graph. In contrast to these works, (1) Our methods discover diversified graph summaries in terms of approximate pattern matching, and (2) We propose an algorithm with parallel scalability guarantee which has not been mentioned in previous works.

Querying using views: View-based query evaluation has been shown to be effective for SPARQL queries [13] and general graph pattern queries [4]. It requires equivalent query rewriting by accessing views defined in the same query language. In contrast, we show that d-summaries can be used as views for graph queries defined by subgraph isomorphism, rather than requiring queries to be in the same semantics. We also develop parallel summarization algorithms. These are not addressed in [4, 13].

2 KNOWLEDGE GRAPH SUMMARIZATION

2.1 Graphs and Summaries

We start with the notions of knowledge graphs and summaries.

Knowledge graphs. A knowledge graph G is a directed labeled graph (V, E, L) with a set of nodes V and a set of edges $E \subseteq V \times V$. Each node $v \in V$ has a label $L(v)$ that may carry the content of v (*e.g.,* type, name, attribute values) as found in knowledge bases and property graphs [9]; and each edge $e \in E$ has a label $L(e)$.

We do not assume a standard schema over G, and our techniques will benefit from such a schema, if exists. Fig. 1 depicts a fraction of a typed knowledge graph. Each entity (*e.g.,*J.Browne) has a label that carries its type (*e.g.,*artist), and connects to other typed entities (*e.g.,*band) via labeled relationships (*e.g.,* collaborated).

Summaries. We review the notions of summaries in [21] below. Given a knowledge graph G, a d-summary P of G is a connected graph pattern (V_P, E_P, L_P), where V_P (resp. $E_P \subseteq V_P \times V_P$) is a set of summary nodes (resp. edges). Each node $u \in V_P$ (resp. edge $e \in E_P$) has a label $L_P(u)$ (resp. $L_P(e)$), and represents a non-empty node set $[u]$ (resp. edge set $[e]$) from G.

d-matching [21]. Given a pattern P and a graph G, a *backward* (resp. *forward*) d-*matching* from P to G is a *nonempty* binary relation $R_d^{\uparrow} \subseteq V_P \times V$ (resp. $R_d^{\downarrow} \subseteq V_P \times V$), where

- $(u, v) \in R_0^{\uparrow}$ and $(u, v) \in R_0^{\downarrow}$ if $L_P(u)$=$L(v)$;
- $(u, v) \in R_d^{\uparrow}$ if $(u, v) \in R_{d-1}^{\uparrow}$, and for every parent u' of u in P, there exists a parent v' of v in G, such that $L_P(u', u)$=$L(v', v)$ (*i.e.,* edges (u', u) and (v', v) have the same edge label), and $(u', v') \in R_{d-1}^{\uparrow}$;

- $(u, v) \in R^{\downarrow}_{d}$ if $(u, v) \in R^{\downarrow}_{d-1}$, and for every child u' of u in P, there exists a child v' of v in G such that $L_P(u, u')$=$L(v, v')$, and $(u', v') \in R^{\downarrow}_{d-1}$.

We define a d-match R_d between P and G as the set of node pairs $\{(u, v)|(u, v) \in R^{\uparrow}_{d} \cap R^{\downarrow}_{d}\}$. We say P is a *d-summary* of G (denoted as $P \sim G$), if for every summary node u and every node $v \in [u]([u] \neq \emptyset)$, $(u, v) \in R_d$. The *base graph* of P in G, denoted as G_P, refers to the subgraph of G induced by the node set $\bigcup_{u \in V_P} [u]$, and the edge set $\bigcup_{e \in E_P} [e]$, for each $u \in V_P$ and $e \in E_P$.

Intuitively, a d-summary P guarantees that G preserves the parent and child relation of each summary node up to hop d. Note that a base graph can be disconnected for a connected summary. Given a knowledge graph G and an integer d, a *summarization* $\mathcal{S}_G$ of G is a set of d-summaries. In practice, additional mapping structures can be used to trace the base graphs for the summaries.

Example 2: Fig. 1 illustrates a summarization of the knowledge graph G, which contains three 2-summaries P_1, P_2, and P_3. The base graph of P_1 contains the entities: $[genre]$={country, punk}, $[film]$={Going Home, Four_Holidays}, $[artist]$={T.McGraw, D.Yoakam, J.Browne}, $[band]$={The_Eagles, Husker_Du,Def_Leppard}. Note that P_1 cannot summarize T.Hanks as the latter has no path to a band as required by P_1. Similarly, P_2 summaries band Def_Leppard and The_Eagles, and their associated country and manager in G, and P_3 summaries You've got a Mail and Sleepless in Seattle, actors T.Hanks and M.Ryan and their countries. □

Summary verification. Given a graph pattern P, a knowledge graph G and an integer d, the *verification* problem is to check if P is a d-summary of G, and if so, identify the *largest* base graph G_P of P in G. In contrast to its counterpart defined by frequent subgraphs (NP-hard), the verification of d-summaries is *tractable.*

Lemma 2.1. *Given a summary P=(V_P, E_P, L_P), an integer d, and a graph G=(V, E, L), it is in $O(|V_P|(|V_P| + |V|)(|E_P| + |E|))$ time to verify if P is a d-summary of G.*

As a proof of Lemma. 2.1, we outline an algorithm, denoted as valiSum, that determines if P=(V_P, E_P, L_P) is a d-summary in polynomial time. The algorithm valiSum first initializes a match set $[u]$=$\{v|(u, v) \in R^{\uparrow}_{0}, v \in V\}$ for each node $u \in V_P$. It then refines the match sets as follows. (1) It first computes the forward d-similarity relation $R^{\downarrow}_{d}$. For each edge $(u', u) \in E_P$, it iteratively removes all the nodes v' in $[u']$ if there exists no child of v' in G such that $(u, v) \in R^{\downarrow}_{i-1}$, for $i \in [1, d]$. This process repeats until no change can be made to $[u]$, for each node u in V_P. (2) It continues to refine the match sets derived from (1), by removing the nodes that do not satisfy the backward d-similarity relation. (3) If for every node $u \in V_P$, $[u] \neq \emptyset$, P is verified as a d-summary. Otherwise, it determines that P is not a d-summary.

The algorithm keeps the following invariants: (1) For any pair $(u, v) \in R^{\uparrow}_{d} \cap R^{\downarrow}_{d}$, $v \in [u]$ when it terminates. (2) If a node v is removed from $[u]$ at any time, then $(u, v) \notin R_d$. Hence it correctly computes the largest R_d and G_P. To see the complexity, observe that it takes $O((|V_P| + |V|)(|E_P| + |E|))$ time to verify forward and backward d-similarity for a single summary node in V_P. Thus the total verification time is in $O(|V_P|(|V_P| + |V|)(|E_P| + |E|))$.

3 DIVERSIFIED SUMMARIZATION

We next introduce a bi-criteria function that captures the quality of summaries in terms of both informativeness and diversity.

Informative Summaries. We are interested at informative summaries that capture more information. This can be measured by an informativeness function defined as:

$$I(P) = \frac{|P|}{b_P} * \mathrm{supp}(P, G)$$

where $|P|$ (the size of P) is defined as the total number of nodes and edges in P, b_p is a size bound to normalize $|P|$, which can be specified as a recognition budget (*i.e.,* the largest summary size a user can understand) [23], and the *support* $\mathrm{supp}(P, G)$ is defined as $\frac{|G_P|}{|G|}$, where $|G_P|$ (resp. $|G|$) is the size (*i.e.,* the total number of nodes and edges) in G_P (resp. G). That is, the informativeness $I(\cdot)$ favors larger summaries that also have higher support.

Example 3: Consider the 2-summaries P_1-P_3 of the graph G (with 45 entities and edges) in Fig. 1. Let the summary size bound b_P=8, we can verify that the size of the base graph $|G_{P_1}|$ is 20. Hence, $\mathrm{supp}(P_1, G)$=$\frac{20}{45}$, and the informativeness of P_1 $I(P_1)$ is $\frac{7}{8} * \frac{20}{45}$=0.39. Similarly, $I(P_2)$=$\frac{6}{8} * \frac{12}{45}$=0.20, and $I(P_3)$=$\frac{6}{8} * \frac{11}{45}$=0.18. □

Diversified Summaries. A second challenge is to *diversify* the summaries in terms of the entities they summarized. We introduce a distance function for summaries.

Distance function. Given summaries P_1 and P_2, their distance is quantified by the distance function diff defined as:

$$\mathrm{diff}(P_1, P_2) = 1 - \frac{|V_{G_{P_1}} \cap V_{G_{P_2}}|}{|V_{G_{P_1}} \cup V_{G_{P_2}}|}$$

where $V_{G_{P_1}} = \bigcup_{u \in V_{P_1}} [u]$ (resp. $V_{G_{P_2}} = \bigcup_{u \in V_{P_2}} [u]$); that is, it measures the Jaccard distance between the set of entities summarized by P_1 and P_2 in their base graphs. One can verify that diff is a metric, *i.e.,* for any three d-summaries P_1, P_2 and P_3, $\mathrm{diff}(P_1, P_2) \leq \mathrm{diff}(P_1, P_3) + \mathrm{diff}(P_2, P_3)$. Here we quantify entity set difference as a more important factor of summary difference. Label/type difference of the entities can also be applied to quantify weighted V_{G_P} in the distance function diff.

Example 4: Consider the 2-summaries P_1-P_3 of the graph G in Fig. 1. The differences are calculated as follows: $\mathrm{diff}(P_1, P_2)$=1-$\frac{2}{14}$=0.86, where they summarize two common entities {The_Eagles, Def_Leppard}). Similarly, $\mathrm{diff}(P_1, P_3)$= 1.00, and $\mathrm{diff}(P_2, P_3)$=0.90. □

Reduced summaries. While the distance function captures the difference of summaries in terms of their base graphs, informative summaries may contain redundant pattern nodes and edges that can be further reduced. This is not discussed in [21].

Example 5: Consider three summaries: P_1 (Example 1), P^1_1, and P^2_1 as illustrated in Fig. 2. We can verify that these three summaries have a same base graph in G (Fig. 1). Although P^1_1 and P^2_1 are larger than P_1, they contain "redundant" nodes (*e.g.,*film and band in P^1_1, and artist in P^2_1) that do not contribute new information, hence should be "reduced" to a concise summary P_1. □

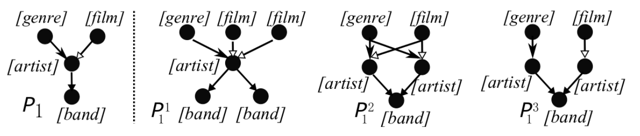


Figure 2: Reduced summary and its "non-reduced" counterparts

Given two d-summaries P_1 and P_2, we say P_1 and P_2 are *equivalent*, denoted as $P_1 \sim P_2$, if there exists a d-matching R_{12} from P_1 to P_2 (denoted as $P_1 \preceq P_2$), and its inverse relation R_{12}^{-1} is a d-matching from P_2 to P_1 (denoted as $P_1 \preceq P_2$). The following result bridges summary equivalence and their support.

Lemma 3.1. *For any graph G and its two d-summaries P_1 and P_2,* $\mathsf{supp}(P_1, G)$=$\mathsf{supp}(P_2, G)$ *if $P_1 \sim P_2$.*

Proof: Given two equivalent summaries P_1=$(V_{P_1}, E_{P_1}, L_{P_1})$ and P_2=$(V_{P_2}, E_{P_2}, L_{P_2})$, it suffice to show that $|G_{P_1}|$=$|G_{P_2}|$, where G_{P_1}=(V_1, E_1, L) (resp. G_{P_2}=(V_2, E_2, L)) refers to the base graph of P_1 (resp. P_2). Denote as the d-matching relation from P_1 (resp. P_2) to G as R_{12} (resp. R_{21}). (1) As $P_1 \preceq P_2$, for every node $u_1 \in V_{P_1}$, there exists a node $u_2 \in V_{P_2}$ such that $(u_1, u_2) \in R_{12}$. As P_2 is a d-summary of G, by definition of d-matching, any match of u_2 in G is also a match of u_1. Thus, $V_2 \subseteq V_1$. Similarly, as $P_2 \preceq P_1$, $V_1 \subseteq V_2$. That is, V_1=V_2. (2) Similarly, we can verify that E_1=E_2. Putting these together, $|G_{P_1}|$=$|G_{P_2}|$. Thus, $\mathsf{supp}(P_2, G) = \mathsf{supp}(P_1, G)$. □

A summary P is a *reduced summary*, if there exists no smaller summary P', such that $P \sim P'$. Indeed, a reduced summary is a smallest representation of its equivalent summaries, without losing the entities it can summarize in G. Better still, the result below verifies that a summary P can be efficiently "reduced".

Lemma 3.2. *Given a summary P=(V_P, E_P, L_P), it is in $O((|V_P| + |E_P|)^2 + |V_P|^2)$ time to compute a reduced summary P_r of P.*

As a proof of Lemma 3.2, we provide a reduction algorithm, denoted as Reduce, as follows. Given a summary P, Reduce (1) computes a d-matching R from P to itself, and (2) identifies all the node pairs (u, v) such that $(u, v) \in R$ and $(v, u) \in R$. The node pairs forms an equivalence relation $R^* \subseteq R$. It then "merges" all the nodes in the same equivalence class to a single node $[u]$, and redirects the edges to $[u]$. This yields a reduced d-summary P_r of P.

Analysis. It is easy to verify that $P_r \preceq P$ and $P \preceq P_r$. We now prove that P_r is the smallest pattern among its equivalent counterparts, by contradiction. Assume there exists a smaller summary P'_r such that $P'_r \sim P$. Then $P'_r \sim P_r$. Denote as the equivalence relation between P'_r and P_r as $R^{*'}$. Then there exists at least two *distinct* nodes u, u' in P_r, and a third node v in P'_r, such that $(u, v) \in R^{*'}$, $(v, u) \in R^{*'}$, $(u', v) \in R^{*'}$, and $(v, u') \in R^{*'}$. Thus, u, u' and v in P belongs to the same equivalent class. Nevertheless, u and u' are not merged in P_r. Thus, either P'_r is not equivalent to P, or $|P_r|$=$|P'_r|$. Either leads to a contradiction.

The above reduction procedure Reduce takes $O(|V_P| + |E_P|)^2)$ time to compute the equivalence relation, and $O(|V_P|^2)$ to perform the reduction. The total cost is hence in $O((|V_P| + |E_P|)^2 + |V_P|^2)$ time. Lemma 3.2 thus follows.

Example 6: Following Example 5, Reduce finds that the two film nodes and two band nodes belong to two equivalent classes in P_1^1, thus reduces P_1^1 to P_1. P_1^2 can also be reduced to P_1 by merging all artist nodes. Note that P_1^3 is a reduced summary but not equivalent to P_1, as artist in P_1 can not be matched with those in P_1^3. □

Diversified Summarization. We now introduce a bi-criteria function F that integrates informativeness $I(\cdot)$ and distance $\mathsf{diff}(\cdot)$ functions. Given a summarization $\mathcal{S}_G$ for a knowledge graph G, the function F is defined as:

$$F(\mathcal{S}_G) = (1 - \alpha) \sum_{P_i \in \mathcal{S}_G} I(P_i) + \frac{\alpha}{\mathsf{card}(\mathcal{S}_G) - 1} \sum_{P_i \neq P_j \in \mathcal{S}_G} \mathsf{diff}(P_i, P_j)$$

where (1) $\mathsf{card}(\mathcal{S}_G)$ refers to the number of summaries it contains; and (2) $\alpha (\in [0, 1])$ is a tunable parameter to trade-off informativeness and diversification. We scale down the second summation (diversification) which has $\frac{\mathsf{card}(\mathcal{S}_G)(\mathsf{card}(\mathcal{S}_G)-1)}{2}$ terms, to balance out the fact that the first summation has $\mathsf{card}(\mathcal{S}_G)$ terms.

Example 7: Set b_p=8 and α=0.1, a top-2 diversified summarization $\mathcal{S}_G$ of G (Fig. 1) is $\{P_1, P_2\}$, with total quality score $F(\mathcal{S}_G)$= $0.9 * (0.39 + 0.20)$+$0.1 * 0.86 = 0.62$. □

Based on the quality metrics, we next introduce a graph summarization problem for knowledge graphs.

Problem statement. Given a knowledge graph G, integers k and d, and a size budget b_p, the *diversified graph summarization* problem is to compute a set of k summaries $\mathcal{S}_G$ of G such that

- each summary in $\mathcal{S}_G$ is a reduced d-summary with size bounded by b_p; and
- the quality function $F(\mathcal{S}_G)$ is maximized.

That is, it finds k reduced summaries that are both informative and diversified. Although desirable, it is (not surprisingly) NP-hard. The hardness can be shown by a reduction from the maximum dispersion problem [8] (a known NP-complete problem).

Despite the hardness, we show that diversified summarization is feasible over large graphs, by providing a *parallel approximation algorithm* in Section 4.

4 PARALLEL DIVERSIFIED SUMMARIZATION

We start with a sequential mining algorithm. Let $\mathcal{S}_G^*$ denote the optimal summarization that maximizes the function F. An ϵ-approximation algorithm returns a summarization $\mathcal{S}_G$, such that $F(\mathcal{S}_G) \geq \frac{F(\mathcal{S}_G^*)}{\epsilon}$ ($\epsilon \geq 1$). We show that diversified summarization is 2-approximable, by presenting such an algorithm.

Approximated Summarization. Given a graph G, integers k and d, and size budget b_p, the algorithm, denoted as approxDis, has the following steps. (1) It invokes a mining algorithm sumGen (G, k, d, b_p) to discover a set C_P of reduced d-summaries. (2) It then invokes a diversification algorithm sumDiv to compute the top-k diversified summaries $\mathcal{S}_G$ from C_P.

Auxiliary structure. Underlying approxDis is the maintenance of a reduced summary lattice $\mathcal{P}$=(V_r, E_r) encodes the generation of d-summaries, where each node $P_r \in V_r$ at level i of $\mathcal{P}$ is a reduced d-summary with i edges, and there exists an edge e_t=$(P_r, P'_r) \in E_r$, if P_r and P'_r are two reduced d-summaries at level i and $i + 1$ ($j \in [1, b_p - 1]$), and P'_r is obtained by adding an edge to P_r.

Procedure sumGen. Procedure sumGen follows conventional pattern mining (*e.g.,* level-wise) to generate summaries with $\mathcal{P}$. The difference is that it uses a validation procedure that guarantees each group of equivalent summaries is verified only once.

For each pattern P, it validates if P is a reduced d-summary in two steps. (1) It first "reduces" P to its reduced counterpart with procedure Reduce in polynomial time (Lemma 2.1). (2) Given a reduced pattern P', it checks whether there exists a reduced summary P_r at level-$|P'|$ such that $P_r \sim P'$. If so, it sets supp(P', G)=supp(P_r, G) *without* verification (Lemma 3.1). Otherwise, it invokes procedure valiSum (Lemma 2.1) to verify P'. It stores supp(P', G) and updates C_P if P' is a valid d-summary.

Procedure sumDiv. Given a set of reduced summaries C_P, procedure sumDiv greedily adds a summary pair $\{P, P'\}$ from C_P to $\mathcal{S}_G$ that maximally improves a function $F'(\mathcal{S}_G)$, defined as $F'(P, P')$= $(1-\alpha)$ $(I(P) + I(P'))$+α $*$diff(P, P'). That is, F' is rounded down from the original function F, which guarantees an approximation ratio. This step is repeated $\lfloor \frac{k}{2} \rfloor$ times to obtain top-k d-summaries $\mathcal{S}_G$. If k is odd, it selects an additional summary P that maximizes $F(\mathcal{S}_G \cup \{P\})$ after $\lfloor \frac{k}{2} \rfloor$ rounds of selection.

Analysis. The correctness of approxDis follows from the correctness of procedure sumGen, and that procedure Reduce and Validate correctly validates all reduced d-summary candidates. Following [8], approxDis simulates an approximation for maximum dispersion that guarantees approximation ratio 2.

For complexity, approxDis takes $O(t_1(G, b_p))$ (the cost of sumGen) + $O(t_2(G, k))$ (the cost of sumDiv) time. Denote as N the total verified patterns in sumGen. (1) By Lemma 2.1 and Lemma 3.2, the verification takes $O(t_1(G, b_p))$ = $O(N * b_p|V||E|)$ time. (2) The diversification sumDiv takes in total $O(t_2(G, k))$=$O(\frac{k}{2}N^2|V|)$ time. Thus, the total cost is in $O(t_1(G, b_p))$+$O(t_2(G, k))$= $O(N * b_p|V||E| + \frac{k}{2}N^2|V|)$ time.

The sequential algorithm is expensive when G is large: both mining and diversification are two intractable processes. We can do better with parallel diversified summarization algorithm.

4.1 Parallel Approximability

We consider a partition strategy $\mathcal{P}$ that constructs a fragmentation $\mathcal{G}$ of G, by distributing G to n workers, where each worker P_i ($i \in [1, n]$) manages its local fraction (a subgraph) of G, denoted as G_i. To characterize the effectiveness of parallel summarization, we introduce a class of *parallel approximation* algorithms.

We first review *parallel scalability* [5, 12]. Consider a "yardstick" sequential algorithm that, given graph G, integer d and size bound b_p, approximately computes the summaries, *e.g.,* algorithm approxDis. Denote the time cost of approxDis as $t(|G|, b_p, k)$. A parallel algorithm A_p is *parallel scalable* if its running time by n processors can be expressed as

$$T(|G|, b_p, k, n) = O(\frac{t(|G|, b_p, k)}{n})$$

Intuitively, it measures the speedup over a sequential algorithm by parallelization. It is a relative measure *w.r.t.* a yardstick sequential algorithm A. A parallel scalable A_p "linearly" reduces the sequential running time of A when n increases.

```
Algorithm paraDis
Input: a fragmented graph $G$, integer $k$, size bound $b_p$;
Output: a set of diversified reduced $d$-summaries.
1.  /* executed at coordinator /*
    set $C_P$:=$\emptyset$; set $\mathcal{S}_G$:=$\emptyset$; lattice $\mathcal{P}$:=$\emptyset$;
    integer $i$:=1; flag newP:=true;
2.  $\mathcal{P}$:=Spawn(0); /* initialize $\mathcal{P}$ with single-node patterns */;
3.  while $i \leq b_p$ and newP do /* superstep $i$ */
4.      set $\Sigma_i$:=Spawn($i$); if $\Sigma_i = \emptyset$ then newP:=false;
5.      if newP then
6.          set $C_{P_i}$ := ParsumGen($\Sigma_i$);/*parallel validation*/
7.          update $\mathcal{P}$; $C_P$:=$C_P \cup C_{P_i}$;
8.          construct work units $M_i$ and distribute $M_i$ to workers;
9.          set $\mathcal{S}_{G_i}$:=ParsumDiv($M_i$); /*parallel diversification*/
10.         update $\mathcal{S}_G$ with $\mathcal{S}_{G_i}$;
11. return $\mathcal{S}_G$;
```

Figure 3: Algorithm paraDis

Parallel approximability. Consider an optimization (*e.g.,* maximization) problem with an optimal solution quantified by a single numerical value x. We say the problem is *parallel* ϵ*-approximable*, if there exists a *parallel scalable* algorithm A_p *w.r.t.* a sequential yardstick algorithm A, and returns a solution $\tilde{x}$ for which $\tilde{x} \leq \epsilon * x$.

We present the main result of this section below.

THEOREM 4.1. *There exists a parallel* 2*-approximable algorithm* w.r.t. *the sequential algorithm* approxDis *that discovers diversified top-k summaries with time cost in* $O(\frac{T(G, b_p, k)}{n})$.

As a proof, we develop a parallel algorithm, denoted as paraDis (illustrated in Fig. 3). It follows Bulk synchronous model and runs in supersteps, where each superstep contains two steps: (1) Parallel verification (denoted as ParsumGen) that "parallelizes" its sequential counterpart sumGen to generate and verify summaries (as in Algorithm approxDis), and (2) Parallel diversification (denoted as ParsumDiv) that "parallelizes" its sequential counterpart sumDiv (as in Algorithm approxDis) to update $\mathcal{S}_G$.

Below we introduce the algorithm paraDis.

4.2 Parallel Diversified Summarization

Overview. Given a fragmented graph $\mathcal{G}$, the algorithm paraDis, shown in Figure 3, executes at most b_p supersteps. It first invokes an operator Spawn(0) to initialize $\mathcal{P}$ with single node patterns (line 2). At each superstep i, it performs the following.

(1) It invokes Spawn(i) to generate a set Σ_i of patterns of size i at a coordinator S_c (line 4). It then invokes ParsumGen to verify the patterns at the workers, in parallel (line 6), and updates $\mathcal{P}$ with new reduced summaries if any (line 7).

(2) It then constructs work units M as pairs of summaries, and distributes M to all the workers following a load balancing strategy (to be discussed, line 8). It invokes ParsumDiv to collect the top-k diversified summary pairs $\mathcal{S}_{G_i}$, computed locally at each worker in parallel (line 9), and update $\mathcal{S}_G$ with the summaries that improve the rounded function $F'(\mathcal{S}_G)$ as in approxDis.

The above steps repeat until b_p supersteps are executed, or no new pattern can be generated, indicated by a flag newP (line 3).

Below we present procedures ParsumGen and ParsumDiv.

Parallel verification. Upon receiving Σ_i from Spawn(i), procedure ParsumGen validates Σ_i in parallel as follows.

(1) At S_c, for each pattern $P \in \Sigma_i$, ParsumGen identifies a verified summary P_r in $\mathcal{P}$ such that P is obtained by adding an edge e to P_r. It then constructs a work unit (P_r, e, j), which encodes a request that "validate if P is a d-summary with the base graph of P_r and edge matches e locally at worker S_j". It then distributes all the work units to their corresponding workers to be validated in parallel, following a workload balancing strategy.

(2) Upon receiving a set of work units, for each work unit (P_r, e, j), each worker S_i performs *incremental validation* for P as the "union" of P_r and e, which (a) issues an on-demand fetching of $e(G_k)$, the local edge matches of e from other workers S_k and (b) verifies P in the graph $P_r(G_j) \cup \bigcup_{k \in [1,n]} e(G_k)$ (*i.e.,* the "union" of local matches $P_r(G_j)$ and all the edge candidates of e) instead of the entire G_j. Each worker stores $P(G_j)$ for the next round of computation.

For each verified P, it constructs a bit vector $P.\mathsf{lvec}_j$ with length $|G_j|$ that encodes the matches of P ($P.\mathsf{lvec}[v]$=1 if node v is a match; similarly for edges) and returns $P.\mathsf{lvec}_j$ in a message M_j.

(3) Upon receiving all the messages M_j, S_c computes $P.\mathsf{lvec}$ by performing "OR" over $P.\mathsf{lvec}_j$ ($i \in [1, n]$), and obtain the support of P. This completes a round of parallel validation.

Parallel Diversification. Given the verified d-summaries C_P so far (including C_{P_i}) (line 7), ParsumDiv updates diversified summaries $\mathcal{S}_G$ in parallel, as follows.

(1) At S_c, for each summary $P \in C_{P_i}$, paraDis constructs a work unit w_P. The work unit w_P consists of (a) P and the vector $P.\mathsf{lvec}$, and (b) a set of summaries $D_P \subseteq C_{P_i}$, as well as their bit vectors, which encodes a request that "computes the distances between P and the summaries in D_P". Given the work units M=$\bigcup_{P \in C_{P_i}} w_P$, it distributes M to all the workers, following a work load balancing strategy (to be discussed, line 8).

(2) Upon receiving a set of work units M_j, for each work unit $w_P \in M_j$, a worker S_j computes the distances diff(P, P') ($P' \in D_P$) by bit vector operations on $P.\mathsf{lvec}$ and $P'.\mathsf{lvec}$. It locally executes a top-k query to find out the local top-k diversified pairs $\mathcal{S}_{G_j}$ that maximize the diversification function F', and returns $\mathcal{S}_{G_j}$ to S_c.

(3) The coordinator S_c collects local top-k pairs from all the workers, and updates $\mathcal{S}_{G_i}$ as $\bigcup_{j \in [1,n]}\{\mathcal{S}_{G_j}\}$ (line 10). It then updates $\mathcal{S}_G$ with the new summary pairs in $\mathcal{S}_{G_i}$.

Load balancing. We partition G with linear deterministic greedy (LDG) scheme [22], which assign a vertex to the partition where it has the most edges. This reduces skewed distribution due to high-degree edges. Algorithm paraDis further uses strategies below.

(1) For each work unit (P_r, e, j) (ParsumGen), $e(G)$ is evenly distributed to all workers. paraDis estimates a "runtime skewness" of $P_r(G)$ at S_j as $|1 - \frac{n*|P_r(G_j)|}{|P_r(G)|}|$. If the skewness is above a threshold, it evenly redistributes $P_r(G)$ to all workers.

(2) For each work unit w_P with a set of summaries D_P, paraDis estimates an upper bound of the diversification cost as $|D_P||V|^2$, and assigns the cost to w_P. It then adopts a greedy strategy following the generalized assignment problem [20] which iteratively assigns a work unit with the smallest cost to a worker with the (dynamically updated) least load. By developing an approximation preserving reduction, we can verify that this algorithm is a 2-approximation [20], and takes $O(|W_j|n \log n)$ time ($|W_j| \le |C_{P_i}|$) for at most $|C_{P_i}|$ validated summaries at superstep i.

Performance Analysis. To see that paraDis is parallel scalable, it suffices to show that parallel verification and diversification are parallel scalable *w.r.t.* their sequential counterparts in approxDis.

<u>*Parallel scalability.*</u> Recall that approxDis incurs two parts of cost $O(t_1(G, b_p))$=$O(N * b_p|V||E|)$, and $O(t_2(G, k))$=$O(\frac{k^2}{2}N^2|V|)$. In accordance, paraDis incurs the cost below.

(1) At superstep i, each worker S_j (a) receives $e(G_k)$ ($k \neq j$) in $O(\frac{|E|}{n})$ time, due to the balanced edge partition of G, (b) sends $e(G_j)$ to other n-1 workers in $O(\frac{n-1|E|}{n})$ time, which is bounded by $O(\frac{|V||E|}{n})$ time as $n \ll |V|$, (c) performs local verification in parallel, in $O(\frac{|\Sigma_i|*b_p|V||E|}{n})$ time, and (d) returns the local matches in parallel in $O(\frac{|V||\Sigma_i|}{n})$ time. As there are b_p supersteps, the total cost is in $O(\frac{N*b_p|V||E|}{n})$=$O(\frac{t_1(G,b_p)}{n})$.

(2) At superstep i, ParsumDiv takes in total $O(\frac{k^2}{2}|M_j||D_P|^2|V|)$ time to computes top-k summary pairs, bounded by $O(\frac{\frac{k^2}{2}|C_{P_i}|^2|V|}{n})$, as all the summaries are from C_{P_i}. The parallel cost of sending top summaries to S_c is in $O(\frac{b_p*k}{n})$. The total parallel diversification cost is thus in $O(\frac{\frac{k^2}{2}N^2|V|}{n})$=$O(\frac{t_2(G,k)}{n})$.

Putting these together, algorithm paraDis takes in total $O(\frac{t_1(G,b_p)}{n})$ + $O(\frac{t_2(G,k)}{n})$ time, hence is parallel scalable *w.r.t.* its sequential counterpart approxDis.

<u>*Approximation.*</u> The quality guarantee of paraDis follows from the invariant that at any superstep i, the set $\mathcal{S}_G$ is a 2-approximation of the optimal diversified summaries from the validated ones. Indeed, it suffices to identify the top-k summaries from ParsumDiv. Thus, paraDis generates $\mathcal{S}_G$ with approximation ratio 2 when terminates, and is a parallel 2-approximation algorithm.

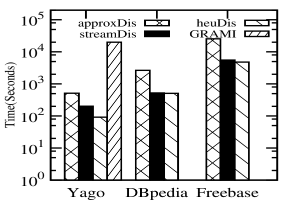
The above analysis completes the proof of Theorem 4.1.

<u>*Remark.*</u> paraDis can be readily extended to parallelize "anytime" summarization (streamDis) [21]. To this end, it only needs to maintain a cache at the coordinator to store (a) a set of summaries, and (b) for each summary, top-k most different summaries (that maximize function diff(·)). It performs parallel verification and diversification to maintain the cached summaries at coordinator, in parallel, and returns the summarization whenever requested.

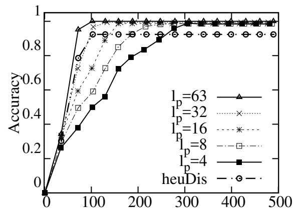
5 EXPERIMENTAL EVALUATION

Using real-life and synthetic data, we evaluated the efficiency of sequential algorithm approxDis, the scalability of parallel algorithm paraDis and the effectiveness of the summaries.

Experimental Setting. We used the following setting.

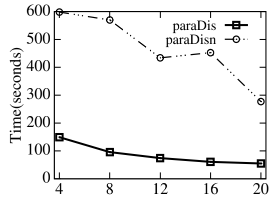


(a) Real-world datasets

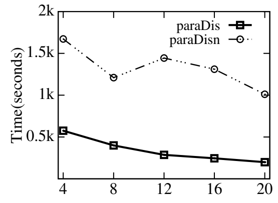


(b) streamDis: quality vs. time(s)

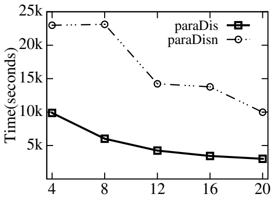
Figure 4: Sequential summarization: Performance



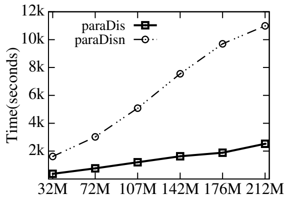
(a) paraDis: Varying n (YAGO)



(b) paraDis: Varying n (DBpedia)



(c) paraDis: Varying n (Freebase)



(d) paraDis: Varying $|G|$ with 20 workers

Figure 5: Scalability of paraDis

Datasets. We use three real-life graphs: (1) *DBpedia*[1] contains 4.86M nodes and 15M edges and 676 labels (*e.g.,*'Settlement', 'Person', 'Building'); (2) *YAGO*[2], a sparser graph compared to *DBpedia* with 1.54M nodes and 2.37M edges, but with more diversified (324343) labels; and (3) *Freebase* (version 14-04-14)[3], with 40.32M nodes, 63.2M edges, and 9630 labels.

We also use *BSBM*[4] e-commerce benchmark to generate synthetic knowledge graphs over products with different types, related vendors, consumers, and views. The generator is controlled by the number of nodes (up to 60M), edges (up to 152M), and labels drawn from an alphabet of 3080 labels.

Algorithms. We implemented the algorithms below, all in Java:

(1) Sequential algorithm approxDis, compared with (a) GRAMI [3] that discovers frequent subgraphs as summaries, where the base graph of a summary is induced by all its isomorphic counterparts in G; (b) stream-style algorithms heuDis and streamDis [21] extended to reduced summaries, where streamDis applies procedure sumDiv to cached (at most l_p) summaries of a stream of verified ones, and heuDis follows streamDis, but simply swaps a summary with a new one that maximize function F;

(2) Parallel algorithm paraDis (including procedures ParsumGen and ParsumDiv), compared with paraDisn, its counterpart without load balancing strategy; and

(3) A query evaluation algorithm extended from [21], which accesses reduced d-summaries instead of d-summaries. It selects and refers to a small set of reduced summaries that best "cover" the query, and fetches entities from the graph only when necessary.

As reduced summaries are a special case of d-summaries, all the techniques and complexity results remain intact. We remark that one can readily plug-in any standard query evaluation algorithms for subgraph queries to the query evaluation framework.

We ran all our experiments on a Linux machine powered by an Intel 2.4 GHz CPU with 128 GB of memory. For the tests of parallel summarization, we used Amazon EC2 r4.large instances, each powered by an Intel 2.8GHz CPU and 16G of memory. We ran each experiment 5 times and report the averaged results.

We next report the details of our findings.

Exp-1: Sequential summarization. We fixed parameter α=0.5 in function F for diversification, k=64, summary size bound b_p=6, and $d = 1$. We set a support threshold θ=0.005. For GRAMI, we carefully tuned its support threshold to allow the generation of patterns with similar label set and size ofapproxDis. We excluded "overly general" (top 2% frequent) labels such as "Thing". For streamDis and heuDis, we report their convergence time [21]. As shown in Fig.4(a), streamDis and approxDis are both orders of magnitude faster than GRAMI. The latter does not run to completion within 10 hours over both DBpedia and Freebase.

Fig. 4(b) verifies that with reduced summaries, streamDis preserves the ability to converge to near-optimal summaries. The additional cost of summary reduction is not significant.

Exp-2: Parallel summarization. We evaluate the parallel algorithm paraDis, compared with paraDisn. We fixed parameter α=0.5 for diversification, k=64, the summary size bound b_p=6, number of hops $d = 1$, and varied the number of workers n from 4 to 20.

We report the performance of paraDis over the real-world datasets in Fig. 5(a)- Fig. 5(c), respectively. (1) paraDis scales well with larger n. The performance of paraDis is improved by 2.6 times from when n is increased from 1 to 4 and 3.3 times for n from 4 to 20. (2) The load balancing strategy improves the performance of paraDisn by 4.4 times on average. (3) It is feasible to summarize large graphs. For example, paraDis takes 200 seconds for paraDis to summarize DBpedia, when n=20, improving its sequential counterpart approxDis by 13 times. (4) In all cases, the accuracy of paraDis (not shown) is almost the same as its sequential counterpart approxDis, due to its parallel approximability.

Using the same setting, we evaluated paraDis with a set of synthetic graphs. The scalability result varying n is consistent with its counterparts over real-world graphs(not shown): paraDis is 3 times faster when n is increased from 4 to 20 for $|G| = (60M, 152M)$ (60 million nodes, 152 million edges). Furthermore, Fig. 5(d) verifies that paraDis scales well with graph size (n=20): it takes 371 seconds for $|G|$=(10M, 22M) and 2.5k seconds for $|G| = (60M, 152M)$.

Exp-3: Case study. We performed two case studies to evaluate the practical application of the knowledge summaries.

Coverage of summaries. We investigate the practical number of diversified summaries needed to "cover" the entity types. We

[1]http://dbpedia.org
[2]http://www.mpi-inf.mpg.de/yago
[3]http://freebase-easy.cs.uni-freiburg.de/dump/
[4]http://wifo5-03.informatik.uni-mannheim.de/bizer/berlinsparqlbenchmark/

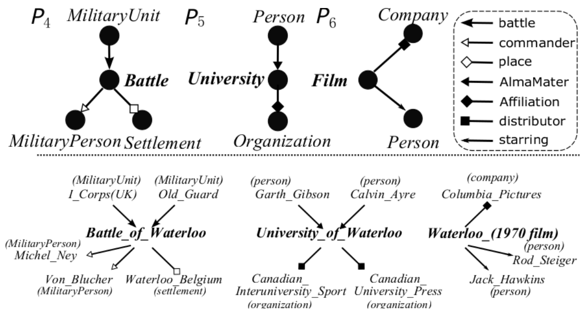


Figure 6: Real-life Summaries: DBpedia.

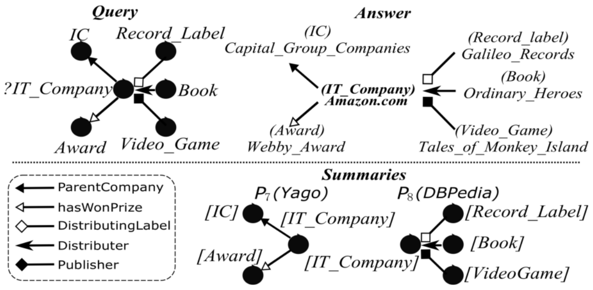


Figure 7: Cross-domain queries over DBpedia **and** Freebase

sampled 50 ambiguous keywords from DBpedia (*e.g.,* "waterloo", "Avatar"), each has on average 4 different types. We found that more diversified summarization (*e.g.,* $\alpha = 0.9$) needs less summaries (*e.g.,* $k = 9$) to cover all the entity types. For all cases, it takes at most 15 summaries to cover all the types. In contrast, most of the summaries from GRAMI are redundant small patterns. It cannot cover the entity types even with 64 summaries.

Three real-life 2-summaries of DBpedia for keyword "waterloo" are shown in Fig. 6. These summaries suggest intermediate keywords as enhanced queries (*e.g.,*Military Person); and can also suggest diversified facts for *e.g.,* Précis queries [23].

Cross-domain queries. We also evaluate how the summaries can be used to support "cross-domain" querying over multiple knowledge bases [2]. We generated 20 cross-domain queries over YAGO and DBpedia. We evaluate the queries by accessing the summaries of YAGO and DBpedia, respectively, and "merges" the matches from each if they have the same URI, to form a complete answer.

We show a query and its answer in Fig. 7. The query finds Award wining IT_Companies with specified products and their parent companies(IC). While YAGO reports parent companies, and DBpedia provides products, a complete answer(Amazon.com) can be found by accessing summaries P_7 and P_8 from YAGO and DBpedia, respectively, integrating the partial answers. This suggests the application of summaries in supporting multi-source querying.

6 CONCLUSIONS

We studied reduced d-summaries and developed sequential and parallel summarization algorithms for large knowledge graphs. Our experimental results verified that our algorithms are feasible, and can significantly reduce the cost of knowledge graph query evaluation. One future work is to enable query suggestion with summaries and use graph summarization for parallel approximate query evaluation [7]. Another topic is to develop summary-based algorithms for more types of analytical queries.

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